Jharkhand University of Technology, Ranchi

B.Tech. 1st Semester Examination, 2018

Subject : Math-I (Calculus and Linear Algebra) **Subject Code : 18107**

Time Allowed : 3 Hours **Full Marks : 70**

Candidates are required to give their answers in their own words as far as practicable.

The figures in the right hand margin indicate full marks.

Answer any five *questions.*

1. There are SEVEN multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which only one is correct. Choose the correct answer: 2×7=14

(i) The value of $\int_0^{\infty} e^{-x^3}\,dx$ is equal to

(a) $\frac{1}{3}\sqrt{\frac{1}{3}}$

(b) $\sqrt{\frac{1}{3}}$

(c) $\frac{1}{3}$

(d) None of the above

2

(ii) The series $\sum_{n=1}^{\infty} \cos\frac{1}{n}$ is

(a) divergent

(b) convergent

(c) oscillatory

(d) None of the above

$0 \times 0 = 0$

(iii) The value of the limit $\log_x \sin x$ is equal to
$x \to 0$

(a) 0

(b) 1

(c) e

(d) None of the above

(iv) Evolute of a curve is the

(a) normals to that curve.

(b) tangents to that curve.

2

(c) locus of the centres of the curvature to that curve.

(d) None of the above

(v) The directional derivative of $\phi = xy^2 + yz^2$ at the point $(2, -1, 1)$ in the direction of the vector $\hat{i} + 2\hat{j} + 2\hat{k}$ is

(a) 14

(b) −9

(c) −3

(d) $\hat{i} + \hat{j} + \hat{k}$

2

(vi) The sum of eigenvalues of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ is equal to

(a) 5

(b) 7

(c) 9

(d) 18

2

(vii) The rank of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ is equal to

(a) 0

(b) 1

(c) 2

(d) 3

2

Or,

1. (a) Find the evolute of the curve $x = a(\theta - \sin\theta), y = a(1 - \cos\theta)$.

(b) Find the surface area of the solid generated by revolving the astroid $x = a\cos^3 t$, $y = a\sin^3 t$ about the x-axis.

(c) Prove that $\int_0^{\pi/2} \sqrt{\cot\theta}\, d\theta = \frac{\sqrt{\frac{3}{4}}\sqrt{\frac{1}{4}}}{2}$. 6+5+3=14

2. (a) Find the value of tan 43° up to 4 places of decimal using Taylor's Theorem.

(b) Show that the diameter of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is equal to the radius of the cone.

(c) Using Lagrange's Mean Value Theorem, prove that $\frac{\cos a\theta - \cos b\theta}{\theta} \leq (b - a)$, if $\theta \neq 0$.

5+5+4=14

3. (a) Expand the function $f(x) = x \sin x$ as a Fourier series in the interval $-\pi < x < \pi$. 3

(b) Test the convergence of the series $\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \cdots \infty, x > 0$. 5

(c) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{\left(1+\frac{1}{n}\right)^{x^2}}$. 3 6+5+3=14

75

48
28
76

4. (a) Find the maximum and minimum distances of the point (3, 4, 12) from the sphere $x^2 + y^2 + z^2 = 4$. 3

(b) If $z = f(u, v)$ and $u = x^2 - y^2, v = 2xy$,

show that $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = 4(u^2 + v^2)^{\frac{1}{2}} \left[\left(\frac{\partial z}{\partial u}\right)^2 + \left(\frac{\partial z}{\partial v}\right)^2\right]$. 7 7+7=14

5. (a) If $x^x y^y z^z = c$, show that at $x = y = z$.

$\frac{\partial^2 z}{\partial x \partial y} = -(x \log ex)^{-1}$, where c is a constant.

(b) Find the half range sine-series for $f(x) = x^2$ in the interval $(0, \pi)$.

(c) Show that the vector $\vec{F} = (y^2 - z^2 + 3yz - 2x)\hat{\imath} + (3xz + 2xy)\hat{\jmath} + (3xy - 2xz + 2z)\hat{k}$ is both solenoidal and irrotational. 5+5+4=14

6. (a) Discuss the consistency of the system and if consistent, solve the equations: 6

$4x - 2y + 6z = 8, \qquad x + y - 3z = -1, \qquad 15x - 3y + 9z = 21$

(b) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ -2 & -1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$

(c) Prove that the matrix $A = {}^1/_3 \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ -2 & 2 & -1 \end{bmatrix}$ is orthogonal. 4 6+4+4=14

7. (a) Verify Caytey-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. Hence, find the inverse of A. 7

(b) Reduce the quadratic form $Q = 3x_1^2 + 5x_2^2 + 3x_3^2 - 2x_1x_2 - 2x_3x_2 + 2x_1x_3$ to the canonical form by orthogonal transformation and specify the matrix of transformation. 7+7=14
